IIT-JEE MAIN
CODE B (2018 Offline)
PAPER - 1 (B.E./B.Tech.)

Questions with SOLUTIONS

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SOLUTIONS Of JEE Main - 2018
MATHEMATICS

OFFLINE MODE - CODE B

(Exam Date : April 08, 2018)

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Q31. If the tangent at (1, 7) to the curve $x^2 = y-6$ touches the circle $x^2+y^2+16x+12y+c=0$ then the value of c is

(1) 85 (2) 95 (3) 195 (4) 185

Sol. Tangent at (1, 7) for $x^2 = y-6$ is : $y-7=2(x-1)$ i.e., $2x-y+5=0$ …(i)

Centre of the circle $x^2+y^2+16x+12y+c=0$ is at C(–8, –6).

Let's now find the coordinates of foot of perpendicular *say* P, of point C to the tangent (i).

Eq. of line CP : $y+6=-\frac{1}{2}(x+8)$ i.e., $y=-\frac{x}{2}-10$ …(ii)

Solving (i) and (ii), we get : $2x+5=-\frac{x}{2}-10$ i.e., $\frac{5x}{2}=-15$ i.e., $x=-6$, so $y=-7$.

Therefore we have P(–6, –7) which will satisfy eq. of circle.

So, $36+49-96-84+c=0$ $\therefore c=95$.

Q32. If L_1 is the line of intersection of the planes $2x-2y+3z-2=0$, $x-y+z+1=0$ and L_2 is the line of intersection of the planes $x+2y-z-3=0$, $3x-y+2z-1=0$, then the distance of the origin from the plane, containing the lines L_1 and L_2, is

(1) $\frac{1}{2\sqrt{2}}$ (2) $\frac{1}{\sqrt{2}}$ (3) $\frac{1}{4\sqrt{2}}$ (4) $\frac{1}{3\sqrt{2}}$

Sol. Clearly L_1 is parallel to $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -2 & 3 \\ 1 & -1 & 1 \end{vmatrix} = \hat{i}+\hat{j}$, and L_2 is parallel to $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 3 & -1 & 2 \end{vmatrix} = 3\hat{i}-5\hat{j}-7\hat{k}$.

Also note that L_1 passes through (0, 5, 4) (*or, find any other point*).

So the required plane is : $\begin{vmatrix} x-0 & y-5 & z-4 \\ 1 & 1 & 0 \\ 3 & -5 & -7 \end{vmatrix} = 0$

i.e., $7x-7y+8z+3=0$ …(i)

Now distance of (i) from (0, 0, 0) is $\frac{|7\times0-7\times0+8\times0+3|}{\sqrt{49+49+64}} = \frac{3}{9\sqrt{2}} = \frac{1}{3\sqrt{2}}$.

Q33. If $\alpha, \beta \in C$ are the distinct roots, of the equation $x^2-x+1=0$, then $\alpha^{101}+\beta^{107}$ is equal to

(1) 1 (2) 2 (3) –1 (4) 0

Sol. Here the roots of $x^2-x+1=0$ are $x=\frac{1+i\sqrt{3}}{2}, \frac{1-i\sqrt{3}}{2}$ i.e., roots are $\alpha=-\omega, \beta=-\omega^2$.

Now $\alpha^{101}+\beta^{107}=(-\omega)^{101}+(-\omega^2)^{107}=-(\omega^{101}+\omega^{214})=-(\omega^{99}\omega^2+\omega^{213}\omega)$

$\therefore \alpha^{101}+\beta^{107}=-(\omega^2+\omega)=-(-1)=1$.

We have used $\omega^2+\omega+1=0$ and, $\omega^3=1$.

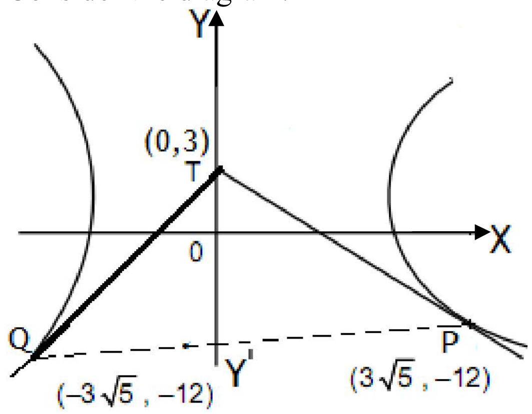
[For $x^3-1=0$, we get $x=1$, ω and, ω^2. We call these roots as Complex cube roots of unity.
Remember, $\omega=\frac{-1+i\sqrt{3}}{2}$, $\omega^2=\frac{-1-i\sqrt{3}}{2}$ represent roots of $x^2+x+1=0$.]

Q34. Tangents are drawn to the hyperbola $4x^2-y^2=36$ at the points P and Q. If these tangents intersect at the point T(0, 3) then the area (in sq. units) of ΔPTQ is

(1) $60\sqrt{3}$ (2) $36\sqrt{5}$ (3) $45\sqrt{5}$ (4) $54\sqrt{3}$

Sol. Given hyperbola is $\frac{x^2}{9}-\frac{y^2}{36}=1$ whose eq. of tangent at $P(x_0, y_0)$ is $\frac{xx_0}{9}-\frac{yy_0}{36}=1$...(i).

Consider the diagram.



For $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, the eq. of tangent at (x_0,y_0) is : $\frac{xx_0}{a^2}-\frac{yy_0}{b^2}=1$

As (i) passes through T(0, 3) so, $\frac{0\times x_0}{9}-\frac{3\times y_0}{36}=1$ i.e., $y_0=-12$ $\therefore x_0=\pm 3\sqrt{5}$

(Note that $P(x_0,y_0)$ is point of contact so, it must satisfy eq. of hyperbola.)

Therefore, we have T(0, 3), $P(3\sqrt{5},-12)$ and $Q(-3\sqrt{5},-12)$.

Hence $ar(PTQ)=$ mag. of $\frac{1}{2}\begin{vmatrix}0 & 3 & 1\\ 3\sqrt{5} & -12 & 1\\ -3\sqrt{5} & -12 & 1\end{vmatrix}=$ mag. of $\frac{1}{2}\left\{-18\sqrt{5}-72\sqrt{5}\right\}=$ mag. of $-45\sqrt{5}$.

Therefore, area of triangle is $45\sqrt{5}$ sq.units.

Q35. If the curves $y^2=6x,\ 9x^2+by^2=16$ intersect each other at right angles, then the value of b is

(1) 4 (2) 9/2 (3) 6 (4) 7/2

Sol. For these curves, we have $\frac{dy}{dx}=\frac{3}{y},\ \frac{dy}{dx}=-\frac{9x}{by}$.

As these curves cut orthogonally so, $\left(\frac{3}{y}\right)\left(-\frac{9x}{by}\right)=-1$ i.e., $\frac{27x}{by^2}=1$

As $y^2=6x$ so, $\frac{27x}{b\times 6x}=1$ $\therefore b=\frac{9}{2}$.

Q36. If the system of linear equations $x+ky+3z=0,\ 3x+ky-2z=0,\ 2x+4y-3z=0$ has a non-zero solution (x, y, z), then $\frac{xz}{y^2}$ is equal to

(1) –30 (2) 30 (3) –10 (4) 10

Sol. As the system of equations has non-zero solution $\therefore \begin{vmatrix}1 & k & 3\\ 3 & k & -2\\ 2 & 4 & -3\end{vmatrix}=0 \Rightarrow k=11$.

Now we have $x+11y+3z=0...(i),\ 3x+11y-2z=0...(ii),\ 2x+4y-3z=0...(iii)$.

Adding (i) and (iii), we get : $3x+15y=0 \Rightarrow x=-5y$.

On putting value of x in (i), we get : $6y+3z=0 \Rightarrow z=-2y$.

Hence $\frac{xz}{y^2}=\frac{(-5y)(-2y)}{y^2}=10$.

Q37. Let $S=\{x\in R: x\geq 0$ and $2\left|\sqrt{x}-3\right|+\sqrt{x}(\sqrt{x}-6)+6=0\}$. Then S

(1) contains exactly two elements (2) contains exactly four elements

(3) is an empty set (4) contains exactly one element

Sol. $2\left|\sqrt{x}-3\right|+\sqrt{x}(\sqrt{x}-6)+6=0 \quad \Rightarrow 2\left|\sqrt{x}-3\right|+(\sqrt{x}-3+3)(\sqrt{x}-3-3)+6=0$

$\Rightarrow 2\left|\sqrt{x}-3\right|+(\sqrt{x}-3)^2-3^2+6=0 \quad \Rightarrow 2\left|\sqrt{x}-3\right|+(\sqrt{x}-3)^2-3=0$

$\Rightarrow (\sqrt{x}-3)^2+2\left|\sqrt{x}-3\right|-3=0 \quad \Rightarrow \left(\left|\sqrt{x}-3\right|+3\right)\left(\left|\sqrt{x}-3\right|-1\right)=0$

Either $\left|\sqrt{x}-3\right|-1=0$ or $\left|\sqrt{x}-3\right|+3=0$

As $\left|\sqrt{x}-3\right|\neq -3 \;\; \therefore \left|\sqrt{x}-3\right|=1$ i.e., $\sqrt{x}-3=\pm 1$ i.e., $\sqrt{x}=4, 2 \qquad \therefore x=16, 4$.

Hence S contains exactly two elements.

Q38. If sum of all the solutions of the equation $8\cos x\left(\cos\left(\frac{\pi}{6}+x\right)\cos\left(\frac{\pi}{6}-x\right)-\frac{1}{2}\right)=1$ in $[0,\pi]$ is $k\pi$, then k is equal to

(1) 8/9 (2) 20/9 (3) 2/3 (4) 13/9

Sol. Here $8\cos x\left(\cos\left(\frac{\pi}{6}+x\right)\cos\left(\frac{\pi}{6}-x\right)-\frac{1}{2}\right)=1 \quad \Rightarrow 8\cos x\left(\cos^2\frac{\pi}{6}-\sin^2 x-\frac{1}{2}\right)=1$

$\Rightarrow 8\cos x\left(\frac{3}{4}-1+\cos^2 x-\frac{1}{2}\right)=1 \quad \Rightarrow 8\cos x\left(\frac{-3+4\cos^2 x}{4}\right)=1$

$\Rightarrow 8\cos^3 x-6\cos x=1$ i.e., $2(4\cos^3 x-3\cos x)=1$ i.e., $\cos 3x=\frac{1}{2} \quad \therefore 3x=\frac{\pi}{3},\frac{5\pi}{3},\frac{7\pi}{3},\ldots$

That implies, $x=\frac{\pi}{9},\frac{5\pi}{9},\frac{7\pi}{9}\in[0,\pi]$. Therefore, the sum of solutions is $\frac{13\pi}{9}$.

Hence $k\pi=\frac{13\pi}{9} \quad \therefore k=\frac{13}{9}$.

Q39. A bag contains 4 red and 6 black balls. A ball is drawn at random from the bag, its colour is observed and this ball along with two additional balls of the same colour are returned to the bag. If now a ball is drawn at random from the bag, then the probability that this drawn ball is red, is

(1) 1/5 (2) 3/4 (3) 3/10 (4) 2/5

Sol. Let E_1 : the first ball drawn is red, E_2 : the first ball drawn is black, and E : the second ball drawn is red

So, $P(E)=P(E_1)P(E|E_1)+P(E_2)P(E|E_2)=\frac{4}{10}\times\frac{6}{12}+\frac{6}{10}\times\frac{4}{12}=\frac{48}{120}=\frac{2}{5}$.

Q40. Let $f(x)=x^2+\frac{1}{x^2}$ and $g(x)=x-\frac{1}{x}$, $x\in R-\{-1,0,1\}$. If $h(x)=\frac{f(x)}{g(x)}$, then the local minimum value of $h(x)$ is

(1) $-2\sqrt{2}$ (2) $2\sqrt{2}$ (3) 3 (4) –3

Sol. Here $h(x)=\frac{f(x)}{g(x)}=\frac{x^2+\frac{1}{x^2}}{x-\frac{1}{x}}=\frac{\left(x-\frac{1}{x}\right)^2+2}{x-\frac{1}{x}}=x-\frac{1}{x}+\frac{2}{x-\frac{1}{x}}$

Let $h(y)=y+\frac{2}{y}$, where $y=x-\frac{1}{x}$.

As $AM\geq GM$ so, $\frac{y+\frac{2}{y}}{2}\geq\sqrt{y\times\frac{2}{y}}$ i.e., $\frac{y+\frac{2}{y}}{2}\geq\sqrt{2}$ i.e., $y+\frac{2}{y}\geq 2\sqrt{2}$

Hence $h(y) \geq 2\sqrt{2}$. Therefore the local minimum value of h (x) is $2\sqrt{2}$.

Q41. Two sets A and B are as under $A = \{(a, b) \in R \times R : |a-5| < 1 \text{ and } |b-5| < 1\}$;
$B = \{(a, b) \in R \times R : 4(a-6)^2 + 9(b-5)^2 \leq 36\}$. Then

(1) $A \cap B = \phi$ (an empty set) (2) Neither $A \subset B$ nor $B \subset A$

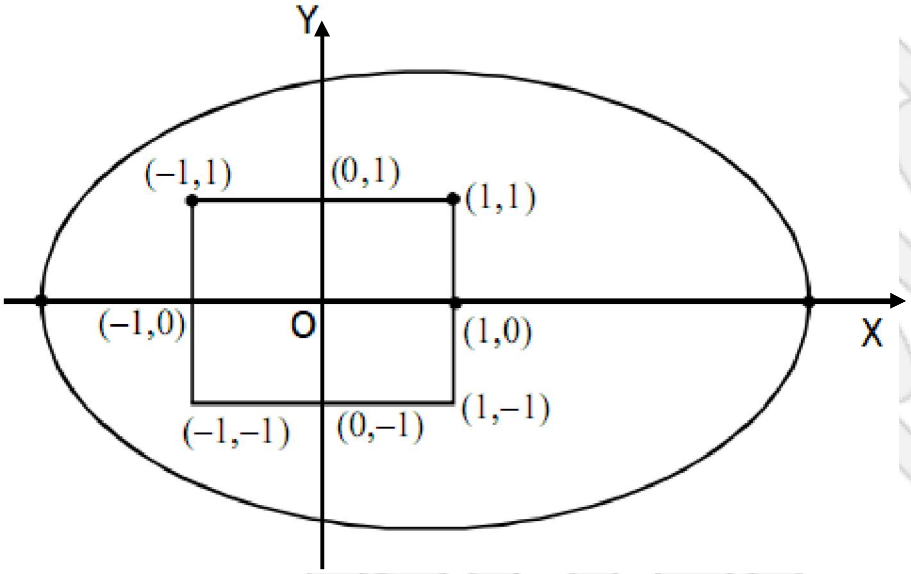
(3) $B \subset A$ (4) $A \subset B$

Sol. Let $a-5 = x,\ b-5 = y$.

Clearly then set A contains all points inside $|x| < 1,\ |y| < 1$.

Similarly B contains all points inside or on the ellipse $\frac{(x-1)^2}{9} + \frac{y^2}{4} = 1$ (as $\frac{(x-1)^2}{9} + \frac{y^2}{4} \leq 1$).

Consider the diagram given below.



As $(\pm 1, \pm 1)$ lies inside the ellipse so, $A \subset B$

Q42. The Boolean expression $\sim(p \vee q) \vee (\sim p \wedge q)$ is equivalent to

(1) q (2) ~q (3) ~p (4) p

Sol. $\sim(p \vee q) \vee (\sim p \wedge q)$

$= (\sim p \wedge \sim q) \vee (\sim p \wedge q)$

$= \sim p \wedge (\sim q \vee q)$

$= \sim p \wedge t = \sim p$.

Q43. Tangent and normal are drawn at P(16, 16) on the parabola $y^2 = 16x$, which intersect the axis of the parabola at A and B, respectively. If C is the centre of the circle through the points P, A and B and $\angle CPB = \theta$, then a value of $\tan\theta$ is

(1) 3 (2) 4/3 (3) 1/2 (4) 2

Sol. For the parabola, eq. of tangent and normal at P(16, 16) are respectively
$2y = x + 16$ and, $y = -2x + 48$.

Here the axis of parabola $y^2 = 16x$ is x-axis. Therefore, A(–16, 0) and B(24, 0).

As C is centre of circle which passes through P, A and B so, CP = CA = CB = radius.

Let C(x, y). Now use distance formula to get 2 equations in x and y. On solving them, we get the coordinates of C(4, 0).

Now slope of CP $= \frac{16-0}{16-4} = \frac{4}{3}$, slope of BP $= \frac{16-0}{16-24} = -2$.

As $\angle CPB = \theta$ so, $\tan\theta = \left|\dfrac{-2-\frac{4}{3}}{1+\frac{4}{3}(-2)}\right| = |2| = 2$

Q44. If $\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (A+Bx)(x-A)^2$, then the ordered pair (A, B) is equal to

(1) (–4, 5) (2) (4, 5) (3) (–4, –5) (4) (–4, 3)

Sol. Apply $C_1 \to C_1 + C_2 + C_3$, $\begin{vmatrix} 5x-4 & 2x & 2x \\ 5x-4 & x-4 & 2x \\ 5x-4 & 2x & x-4 \end{vmatrix} = (A+Bx)(x-A)^2$

So, $(5x-4)\begin{vmatrix} 1 & 2x & 2x \\ 1 & x-4 & 2x \\ 1 & 2x & x-4 \end{vmatrix} = (A+Bx)(x-A)^2$

Also $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$, $(5x-4)\begin{vmatrix} 1 & 2x & 2x \\ 0 & -x-4 & 0 \\ 0 & 0 & -x-4 \end{vmatrix} = (A+Bx)(x-A)^2$

So, $(5x-4)(x+4)^2\begin{vmatrix} 1 & 2x & 2x \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{vmatrix} = (A+Bx)(x-A)^2 \quad \Rightarrow (5x-4)(x+4)^2 = (A+Bx)(x-A)^2$

On comparing, we get : $A = -4,\ B = 5$.

Q45. The sum of the coefficients of all odd degree terms in the expansion of $\left(x+\sqrt{x^3-1}\right)^5 + \left(x-\sqrt{x^3-1}\right)^5$, $(x > 1)$ is

(1) 1 (2) 2 (3) –1 (4) 0

Sol. $\left(x+\sqrt{x^3-1}\right)^5 + \left(x-\sqrt{x^3-1}\right)^5 = 2\left[{}^5C_0x^5 + {}^5C_2x^3(x^3-1) + {}^5C_4x(x^3-1)^2\right]$

$\Rightarrow \quad = 2\left[x^5 + 10(x^6-x^3) + 5(x^7-2x^4+x)\right] = 2\left[x^5 + 10(x^6-x^3) + 5(x^7-2x^4+x)\right]$

$\Rightarrow \quad = 2\left[x^5 + 10x^6 - 10x^3 + 5x^7 - 10x^4 + 5x\right]$

Hence, sum of odd degree terms coefficients $= 2(5+1-10+5) = 2$.

Q46. Let $a_1, a_2, a_3, \ldots, a_{49}$ be in A.P. such that $\sum_{k=0}^{12} a_{4k+1} = 416$ and $a_9 + a_{43} = 66$. If $a_1^2 + a_2^2 + \ldots + a_{17}^2 = 140m$, then m is equal to

(1) 34 (2) 33 (3) 66 (4) 68

Sol. Let ‘a’ and ‘d’ be the first term and common difference of the A.P. respectively.

Here $a_9 + a_{43} = 66$ i.e., $2a + 50d = 66$ i.e., $a + 25d = 33$...(i)

And $\sum_{k=0}^{12} a_{4k+1} = 416$ i.e., $a_1 + a_5 + a_9 + a_{13} + \ldots + a_{49} = 416 \qquad \Rightarrow 13a + d(4+8+12+\ldots+48) = 416$

$\Rightarrow 13a + 4d\left(\dfrac{12\times 13}{2}\right) = 416 \qquad \Rightarrow 13(a+24d) = 416 \qquad \Rightarrow a + 24d = 32$...(ii)

Solving (i) and (ii), we get : a = 8, d = 1.

Now $a_1^2 + a_2^2 + ... + a_{17}^2 = 140m$ i.e., $8^2 + 9^2 + 10^2 + ... + 24^2 = 140m$

i.e., $\left(1^2 + 2^2 + ... + 7^2 + 8^2 + 9^2 + 10^2 + ... + 24^2\right) - \left(1^2 + 2^2 + ... + 7^2\right) = 140m$

$\Rightarrow \dfrac{24 \times 25 \times 49}{6} - \dfrac{7 \times 8 \times 15}{6} = 140m \qquad \therefore m = 34$.

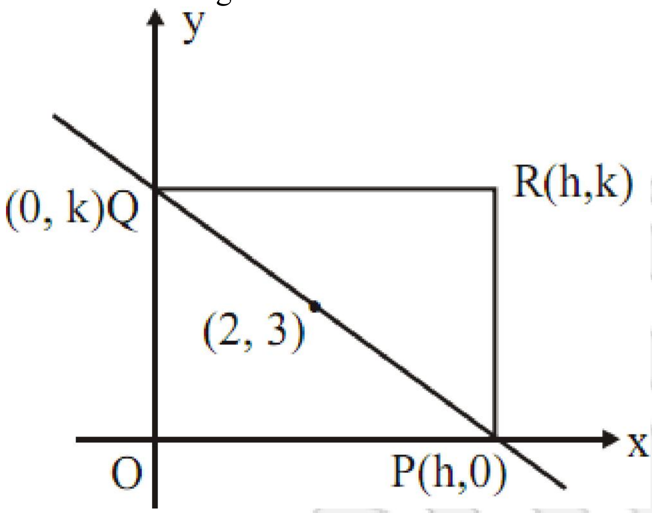
Q47. A straight line through a fixed point (2, 3) intersects the coordinate axes at distinct points P and Q. If O is the origin and the rectangle OPRQ is completed, then the locus of R is

(1) $3x + 2y = xy$ (2) $3x + 2y = 6xy$ (3) $3x + 2y = 6$ (4) $2x + 3y = xy$

Sol. Let R(h, k). Clearly P(h, 0) and Q(0, k).

Equation of line PQ is : $\dfrac{x}{h} + \dfrac{y}{k} = 1 \ldots(i)$

Consider the diagram.



As (i) passes through (2, 3) so, $\dfrac{2}{h} + \dfrac{3}{k} = 1$ i.e., $3h + 2k = hk$

Hence locus of R(h, k) is $3x + 2y = xy$.

Q48. The value of $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 x}{1 + 2^x} dx$ is

(1) 4π (2) $\dfrac{\pi}{4}$ (3) $\dfrac{\pi}{8}$ (4) $\dfrac{\pi}{2}$

Sol. Let $I = \displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 x}{1 + 2^x} dx \ldots(i)$

$$\Rightarrow I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 x}{1 + 2^{-x}} dx = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 2^x \times \frac{\sin^2 x}{1 + 2^x} dx \ldots(ii)$$

Adding (i) and (ii), we get : $2I = \displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 x dx = 2\int_0^{\frac{\pi}{2}} \sin^2 x dx \quad \Rightarrow I = \int_0^{\frac{\pi}{2}} \sin^2 x dx$

$$\Rightarrow I = \frac{1}{2}\int_0^{\frac{\pi}{2}} (1 - \cos 2x) dx = \frac{1}{2}\left[x - \frac{1}{2}\sin 2x\right]_0^{\frac{\pi}{2}} = \frac{1}{2}\left[\left(\frac{\pi}{2} - 0\right) - 0\right] \qquad \therefore I = \frac{\pi}{4}.$$

Q49. Let $g(x)=\cos x^2$, $f(x)=\sqrt{x}$ and $\alpha, \beta\ (\alpha<\beta)$ be the roots of the quadratic equation $18x^2-9\pi x+\pi^2=0$. Then the area (in sq. units) bounded by the curve $y=(gof)(x)$ and the lines $x=\alpha$, $x=\beta$ and $y=0$, is

(1) $\frac{1}{2}(\sqrt{3}-\sqrt{2})$ (2) $\frac{1}{2}(\sqrt{2}-1)$ (3) $\frac{1}{2}(\sqrt{3}-1)$ (4) $\frac{1}{2}(\sqrt{3}+1)$

Sol. For $18x^2-9\pi x+\pi^2=0 \quad \Rightarrow (6x-\pi)(3x-\pi)=0 \quad \therefore x=\frac{\pi}{6},\frac{\pi}{3}$.

Let $\alpha=\frac{\pi}{6}$ and, $\beta=\frac{\pi}{3}$.

Now $y=(gof)(x)=g(f(x))=g(\sqrt{x})=\cos x$.

So, area $\int_{\pi/6}^{\pi/3}\cos x\,dx=[\sin x]_{\pi/6}^{\pi/3}=\frac{\sqrt{3}-1}{2}$.

Q50. For each $t\in R$, let $[t]$ be the greatest integer less than or equal to t. Then

$$\lim_{x\to 0^+} x\left(\left[\frac{1}{x}\right]+\left[\frac{2}{x}\right]+\ldots+\left[\frac{15}{x}\right]\right)$$

(1) is equal to 120 (2) does not exist (in R)

(3) is equal to 0 (4) is equal to 15

Sol. Note that $\left[\frac{k}{x}\right]=\frac{k}{x}-\left\{\frac{k}{x}\right\}$, where $\{.\}$ represents the fractional part function.

Therefore, let $y=\lim_{x\to 0^+} x\left(\left[\frac{1}{x}\right]+\left[\frac{2}{x}\right]+\ldots+\left[\frac{15}{x}\right]\right)=\lim_{x\to 0^+} x\left(\frac{1}{x}-\left\{\frac{1}{x}\right\}+\frac{2}{x}-\left\{\frac{2}{x}\right\}+\ldots+\frac{15}{x}-\left\{\frac{15}{x}\right\}\right)$

$\Rightarrow \quad =\lim_{x\to 0^+} x\left(\frac{1}{x}+\frac{2}{x}+\ldots+\frac{15}{x}\right)-\lim_{x\to 0^+} x\left(\left\{\frac{1}{x}\right\}+\left\{\frac{2}{x}\right\}+\ldots+\left\{\frac{15}{x}\right\}\right)$

As $0\le\left\{\frac{k}{x}\right\}<1 \Rightarrow 0\le x\left\{\frac{k}{x}\right\}<x$

$\therefore y=\lim_{x\to 0^+} x\left(\frac{1+2+\ldots+15}{x}\right)-0 \qquad \therefore y=1+2+\ldots+15=\frac{15(15+1)}{2}=120$.

Q51. If $\sum_{i=1}^{9}(x_i-5)=9$ and $\sum_{i=1}^{9}(x_i-5)^2=45$, then the standard deviation of the 9 items $x_1, x_2, \ldots, x_9$ is

(1) 2 (2) 3 (3) 9 (4) 4

Sol. The S.D. of (x_i-5) is, $\sigma=\sqrt{\frac{\sum_{i=1}^{9}(x_i-5)^2}{9}-\left(\frac{\sum_{i=1}^{9}(x_i-5)}{9}\right)^2}=\sqrt{\frac{45}{9}-\left(\frac{9}{9}\right)^2}=2$.

As S.D. remains unchanged if observations are added/subtracted with/by a fixed constant so, the required S.D. of given 9 items is also 2.

Q52. The integral $\int\frac{\sin^2 x\cos^2 x}{(\sin^5 x+\cos^3 x\sin^2 x+\sin^3 x\cos^2 x+\cos^5 x)^2}dx$ is equal to

(1) $\frac{1}{1+\cot^3 x}+C$ (2) $-\frac{1}{1+\cot^3 x}+C$ (3) $\frac{1}{3(1+\tan^3 x)}+C$ (4) $-\frac{1}{3(1+\tan^3 x)}+C$

Sol. Let $I=\int \frac{\sin^2 x\cos^2 x}{(\sin^5 x+\cos^3 x\sin^2 x+\sin^3 x\cos^2 x+\cos^5 x)^2}dx$

$\Rightarrow I=\int \frac{\sin^2 x\cos^2 x}{\{(\sin^2 x+\cos^2 x)(\sin^3 x+\cos^3 x)\}^2}dx=\int \frac{\sin^2 x\cos^2 x}{(\sin^3 x+\cos^3 x)^2}dx$

$\Rightarrow I=\int \frac{\tan^2 x\sec^2 x}{(\tan^3 x+1)^2}dx$ Put $\tan^3 x+1$ $\Rightarrow \tan^2 x\sec^2 x dx=\frac{dt}{3}$

$\therefore I=\frac{1}{3}\int \frac{1}{t^2}dt=-\frac{1}{3t}+C=-\frac{1}{3(\tan^3 x+1)}+C$

Q53. Let $S=\{t\in R: f(x)=|x-\pi|.(e^{|x|}-1)\sin|x|$ is not differentiable at $t\}$. Then the set S is equal to

(1) $\{\pi\}$ (2) $\{0, \pi\}$ (3) ϕ (an empty set) (4) $\{0\}$

Sol. We have $f(x)=|x-\pi|.(e^{|x|}-1)\sin|x|$

Differentiability at $x=0$:

$Rf'(0)=\lim_{x\to0^+}\frac{|x-\pi|.(e^{|x|}-1)\sin|x|-0}{x-0}=\lim_{h\to0}\frac{|h-\pi|.(e^{|h|}-1)\sin|h|}{h}=0$

$Lf'(0)=\lim_{x\to0^-}\frac{|x-\pi|.(e^{|x|}-1)\sin|x|-0}{x-0}=\lim_{h\to0}\frac{|-h-\pi|.(e^{|-h|}-1)\sin|-h|}{-h}=0$

As LHD = RHD so, f is differentiable at $x=0$.

Differentiability at $x=\pi$:

$Rf'(\pi)=\lim_{x\to\pi^+}\frac{|x-\pi|.(e^{|x|}-1)\sin|x|-0}{x-\pi}=\lim_{h\to0}\frac{|\pi+h-\pi|.(e^{|\pi+h|}-1)\sin|\pi+h|}{\pi+h-\pi}$

$\Rightarrow Rf'(\pi)=\lim_{h\to0}\frac{|h|.(e^{|\pi+h|}-1)\sin|\pi+h|}{h}=0$

$Lf'(\pi)=\lim_{x\to\pi^-}\frac{|x-\pi|.(e^{|x|}-1)\sin|x|-0}{x-\pi}=\lim_{h\to0}\frac{|\pi-h-\pi|.(e^{|\pi-h|}-1)\sin|\pi-h|}{\pi-h-\pi}$

$\Rightarrow Lf'(\pi)=\lim_{h\to0}\frac{|-h|.(e^{|\pi-h|}-1)\sin|\pi-h|}{h}=0$

As LHD = RHD so, f is differentiable at $x=\pi$.

So clearly $f(x)$ is differentiable at all value of x.

Hence, S is an empty set, ϕ (an empty set).

Q54. Let $y=y(x)$ be the solution of the differential equation $\sin x\frac{dy}{dx}+y\cos x=4x,\ x\in(0,\pi)$. If $y\left(\frac{\pi}{2}\right)=0$, then $y\left(\frac{\pi}{6}\right)$ is equal to

(1) $-\frac{8}{9}\pi^2$ (2) $-\frac{4}{9}\pi^2$ (3) $\frac{4}{9\sqrt{3}}\pi^2$ (4) $-\frac{8}{9\sqrt{3}}\pi^2$

Sol. Here $\frac{dy}{dx}+\cot x\ y=4x\operatorname{cosec} x$

As I.F. $=e^{\int\cot x dx}=e^{\log\sin x}=\sin x$

So, the solution is : $\sin x\ y=\int 4x\operatorname{cosec} x\times\sin x dx+C$ $\Rightarrow y\sin x=2x^2+C$

As $y\left(\frac{\pi}{2}\right)=0$ so, $0\sin\frac{\pi}{2}=2\left(\frac{\pi}{2}\right)^2+C$ i.e., $C=-\frac{\pi^2}{2}$

Therefore, $y\sin x=2x^2-\frac{\pi^2}{2}$

For $y\left(\frac{\pi}{6}\right)$, $y\sin\frac{\pi}{6}=2\left(\frac{\pi}{6}\right)^2-\frac{\pi^2}{2}$ $\quad\Rightarrow y\times\frac{1}{2}=2\times\frac{\pi^2}{36}-\frac{\pi^2}{2}$

$\therefore y=\frac{\pi^2}{9}-\pi^2=-\frac{8\pi^2}{9}$.

Q55. Let $\vec{u}$ be a vector coplanar with the vectors $\vec{a}=2\hat{i}+3\hat{j}-\hat{k}$ and $\vec{b}=\hat{j}+\hat{k}$. If $\vec{u}$ is perpendicular to $\vec{a}$ and $\vec{u}.\vec{b}=24$, then $|\vec{u}|^2$ is equal to

(1) 256 (2) 84 (3) 336 (4) 315

Sol. Let $\vec{u}=x\hat{i}+y\hat{j}+z\hat{k}$.

Now $\vec{u}.\vec{a}=0\Rightarrow 2x+3y-z=0$...(i), $\vec{u}.\vec{b}=24\Rightarrow y+z=24$...(ii)

As $\vec{u}$, $\vec{a}$ and $\vec{b}$ are coplanar so, $[\vec{u}\ \vec{a}\ \vec{b}]=0$

$$\therefore\begin{vmatrix}x & y & z\\ 2 & 3 & -1\\ 0 & 1 & 1\end{vmatrix}=0 \qquad\Rightarrow 4x-2y+2z=0\text{...(iii)}$$

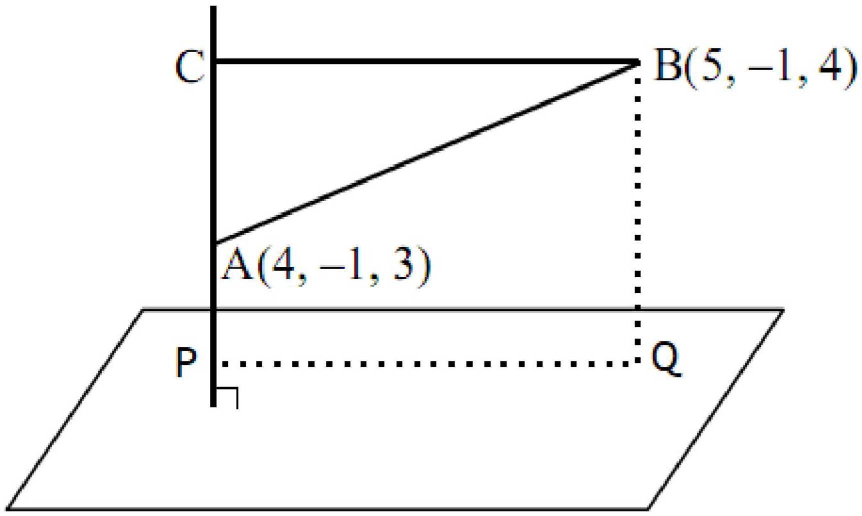
Solving (i), (ii) and (iii), we get : $x=-4,\ y=8,\ z=16$.

Therefore, $\vec{u}=-4\hat{i}+8\hat{j}+16\hat{k}$ $\quad\therefore|\vec{u}|^2=16+64+256=336$.

Q56. The length of the projection of the line segment joining the points (5, –1, 4) and (4, –1, 3) on the plane $x+y+z=7$ is

(1) $\frac{1}{3}$ (2) $\sqrt{\frac{2}{3}}$ (3) $\frac{2}{\sqrt{3}}$ (4) $\frac{2}{3}$

Sol. Normal to the plane, $x+y+z=7$ is $\vec{n}=\hat{i}+\hat{j}+\hat{k}$ which is along $\overrightarrow{PC}$. Also $\overrightarrow{AB}=\hat{i}+\hat{k}$.



Here $AC=$ Length of projection of $\overrightarrow{AB}$ on $\vec{n}=\left|\overrightarrow{AB}.\hat{n}\right|=\frac{2}{\sqrt{3}}$

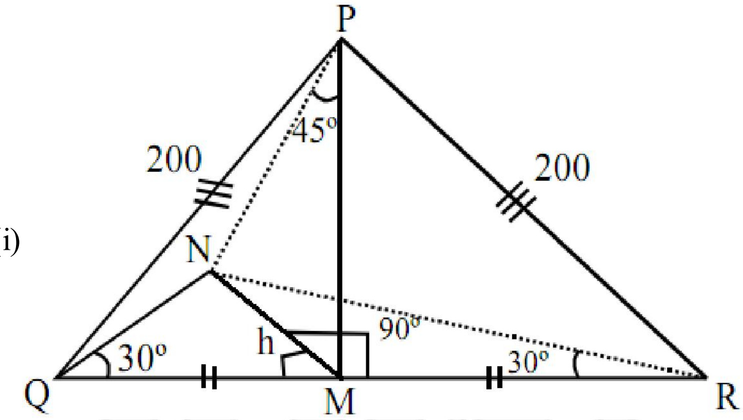
Now $PQ=BC=\sqrt{AB^2-AC^2}=\sqrt{2-\frac{4}{3}}=\sqrt{\frac{2}{3}}$ (We've used Pythagoras Th. in ΔABC.

Hence, required length of projection on the plane is $PQ = \sqrt{\frac{2}{3}}$.

Q57. PQR is triangular park with PQ = PR = 200 m. A T.V. tower stands at the mid-point of QR. If the angles of elevation of the top of the tower at P, Q and R are respectively $45^\circ, 30^\circ$ and 30°, then the height of the tower (in m) is

(1) $100\sqrt{3}$ (2) $50\sqrt{2}$ (3) 100 (4) 50

Sol. Let height of tower MN is 'h'.



In ΔQMN,

$$\frac{MN}{QM} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$\Rightarrow QM = \sqrt{3}h = MR$...(i)

Also in ΔPMR, $PM = \sqrt{200^2 - 3h^2}$ …(i)

(Using Pythagoras Theorem)

Now in ΔMNP,

$$\frac{MN}{PM} = \tan 45^\circ \Rightarrow MN = PM = h \ldots\text{(ii)}$$

By (i) and (ii), we get :

$h = \sqrt{200^2 - 3h^2}$ $\therefore h = 100$ m.

Q58. From 6 different novels and 3 different dictionaries, 4 novels and 1 dictionary are to be selected and arranged in a row on a shelf so that the dictionary is always in the middle. The number of such arrangements is

(1) at least 500 but less than 750 (2) at least 750 but less than 1000

(3) at least 1000 (4) less than 500

Sol. The number of ways of selecting 4 novels from 6 novels is ${}^6C_4$. Also the number of ways of selecting 1 dictionary from 3 dictionaries is ${}^3C_1$.

Required number of arrangement $= {}^6C_4 \times {}^3C_1 \times 4! = \frac{6\times5}{2} \times 3 \times 4 \times 3 \times 2 = 1080$

There the number of arrangement is at least 1000.

Q59. Let A be the sum of the first 20 terms and B be the sum of the first 40 terms of the series $1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + \ldots$. If $B - 2A = 100\lambda$, then λ is equal to

(1) 464 (2) 496 (3) 232 (4) 248

Sol. Here $A = 1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + \ldots + 2.20^2$

$$\Rightarrow A = \left(1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + \ldots + 20^2\right) + \left(2^2 + 4^2 + 6^2 + \ldots + 20^2\right)$$

$$\left[\text{For } 2^2 + 4^2 + \ldots + 20^2,\ a_n = (2n)^2 = 4n^2 \ \therefore \sum_{n=1}^{10} a_n = 4\sum_{n=1}^{10} n^2 = 4 \times \frac{10(10+1)(2\times10+1)}{6}\right]$$

$$\Rightarrow A = \frac{20\times21\times41}{6} + 4\times\frac{10\times11\times21}{6} = \frac{20\times21}{6}\{41+22\} = 4410$$

Also $B = 1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + \ldots + 2.40^2$

$$\Rightarrow B = \left(1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + \ldots + 40^2\right) + \left(2^2 + 4^2 + 6^2 + \ldots + 40^2\right)$$

$$\Rightarrow B = \frac{40\times41\times81}{6} + 4\times\frac{20\times21\times41}{6}$$

$$\Rightarrow B = \frac{20\times41}{6}\left(2\times81 + 4\times21\right) = 33620$$

Now $B - 2A = 100\lambda \Rightarrow 33620 - 2\times4410 = 100\lambda$ $\therefore \lambda = 248$.

Q60. Let the orthocentre and centroid of a triangle be A(–3, 5) and B(3, 3) respectively. If C is the circumcentre of this triangle, then the radius of the circle having line segment AC as diameter, is

(1) $3\sqrt{\frac{5}{2}}$ (2) $\frac{3\sqrt{5}}{2}$ (3) $\sqrt{10}$ (4) $2\sqrt{10}$

Sol. Here orthocentre and centroid of a triangle is at A(–3, 5) and B(3, 3) respectively. Let C be the circumcentre.

As centroid divides orthocentre and circumcentre in 2 : 1 so, AB : BC = 2 : 1.

Now $B(3,3) = B\left(\frac{2x-3}{2+1}, \frac{2y+5}{2+1}\right)$ i.e., $x = 6,\ y = 2$.

So, the coordinates of circumcentre is C(6, 2).

Now required radius $= \frac{AC}{2} = \frac{\sqrt{(6+3)^2+(2-5)^2}}{2} = \frac{\sqrt{81+9}}{2} = \frac{3}{2}\sqrt{10} = 3\sqrt{\frac{10}{4}} = 3\sqrt{\frac{5}{2}}$.

◘

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(Along with other Entrance Exams)

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• RUSTAM RAZA	JEE-Main Qualified
• RISHABHA SINGH	JEE-Main Qualified (NIT-Calicut)
• ANKIT KUMAR	JEE-Main & IPU Qualified
• AKASH SINHA	IPU (Amity Institute Of Technology)
• SOURABH JHA	IPU (Amity Institute Of Technology)

IIT JEE-2014 QUALIFIERS

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• AKSHAY	JEE-Main Qualified
• VANSHIKA SINGH	JEE-Main Qualified
• SOURABH SUDHAKAR	IPU Qualified

IIT JEE-2013 QUALIFIERS

(Along with other Entrance Exams)

• DHAWAL JAIN	JEE-Main & Advanced Qualified - All India Rank : 1264 (IIT-Delhi)
• LAKSHYA SHARMA	IPU (Amity Institute Of Technology)
• PRAGATI	IPU (Amity Institute Of Technology)

OUR SUCCESS GRAPH

*Results of 2018 is awaited.


results

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Chhawla

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School, Dwarka

MOHIT KAUSHIK

Marks 98

Roll No. 9730351
Govt. Sarvodaya
Co-ed School, Najafgarh

SHIVANGI VERMA

Marks 97

Roll No. 9730041
Sarvodaya Kanya Vidyalaya,
Dharam Pura

ANAND VERMA

Marks 95

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Green View Public School,
Najafgarh

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CRPF Public School,
Rohini

RAJEEV YADAV

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AKRITI

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Marks 95

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HARSH

Marks 93

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ABHISHEK MISHRA

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Marks 95

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Delhi Public School
Dwarka
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Najafgarh
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AKASH PUNDIR

Marks 95

Roll No. 9198859
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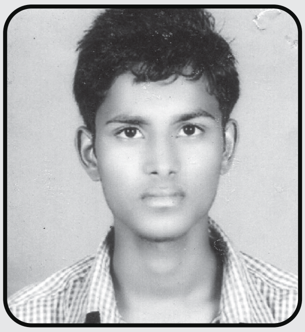
ASHWINI KUMAR



Marks 95

Roll No. 9198825
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Najafgarh
(School Code : 65692)

MAYANK YADAV

Marks 95

Roll No. 9198834
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Mohak Sahu

Marks 96

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Sagar Public School

Rachna Sinha

Marks 95

Roll No. 9170307
Spring Meadows, Dwarka Mor

Bhavya Arora

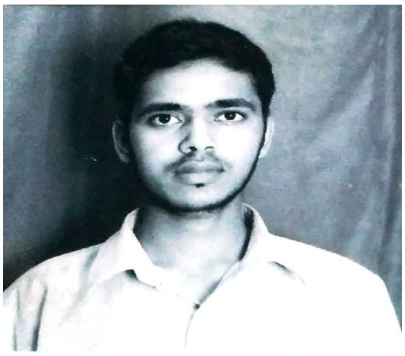
Marks 95

Roll No. 9199877
Birla Vidya Niketan

Rustam Raza

Marks 95

Roll No. 9703080
SBV, No.2, Tilak Nagar



Ankit Kumar

Marks 95

Roll No. 9188601
Shiksha Bharati, , Dwarka

Akash Sinha

Marks 95

Roll No. 9170380
Spring Meadows, Dwarka Mor

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24 Students of our Classes scored between
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Roll No 1669646
Sagar Public School

Suyash Gupta
MARKS 99
Roll No 1669710
Sagar Public School

Nischal Agrawal
MARKS 98
Roll No 1669733
Sagar Public School

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Roll No 9102281
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Akbar Jamal
MARKS 96
Roll No 4608999
SSVM

Saurabh Sudhakar
MARKS 95
Roll No 9720776
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Nidhi
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Nath valley School

Arpit Agarwal
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Rishabha Singh
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New Way School

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MARKS 95
Roll No 5652312
Amity Intl. School, Noida

Kavi Bharathi R
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Roll No 4609007
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SANDEEP Kr.

Marks 96

Roll No. 9702127

SV, Dharam Pura (2011)

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Marks 96

Roll No. 9705302

SBV, No.2, Tilak Nagar

RENU YADAV

Marks 95

Roll No. 9757968

GGSSS, No.1

NIKITA SAXENA

Marks 95

Roll No. 9757952

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PRIYA MISHRA

Marks 95

Roll No. 9188518

KV, Chhawla

POOJA SHARMA

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SKV, Dharam Pura